ON THE PHASE OF THE OFF-SHELL SCATTERING AMPLITUDES

V.A.Petrov

Division of Theoretical Physics, IHEP, Protvino, Russia

Abstract

We prove that the phase of the partial wave amplitude does not depend on the off-shell 4-momentum squared of one of the scattered particles.

Let $T_l(s)$ is the partial wave amplitude for elastic scattering with angular momentum l and c.m.s. energy $\sqrt{s}$. Assuming that there is no unphysical thresholds we consider "elastic region" in s, where unitarity takes a simple form

$$Im\, T_l(s) = |T_l(s)|^2 \,. \tag{1}$$

Let now one of the four interacting particles is off-shell, and denote the corresponding amplitude by $T_{l*} \equiv T_l(s, q^2 \neq m^2)$. If two particles (one final and one initial) are off mass-shell then T_{l**} is the corresponding amplitude. For simplicity we take all off-shell momenta below the mass-shell.

In this case for the same "elastic region" one can get from unitarity that

$$Im\, T_{l**} = |T_{l*}|^2 \,, \tag{2}$$

$$ImT_{l*} = ReT_l ReT_{l*} + ImT_l ImT_{l*}. \tag{3}$$

In terms of the following parametrisation

$$T_l = r_l e^{i\varphi_l}, \quad T_{l*} = r_{l*} e^{i\varphi_{l*}}, \quad T_{l**} = r_{l**} e^{i\varphi_{l**}}$$

we have from Eqs.(1), (2) and (3)

$$\sin\varphi_l = r_l,$$

$$r_{**l} \sin\varphi_{l**} = r_{l*}^2,$$

$$\sin\varphi_{l*} = \sin^2\varphi_l \sin\varphi_{l*} + \sin\varphi_l \cos\varphi_l \cos\varphi_{l*},$$

whereof we came to the equality:

$$\varphi_{l*} = \varphi_l.$$

That is the main result of this note:

onefold off-shell extension of the partial wave scattering amplitude in the "elastic region" in the c.m.s. energy does not change the phase. In particular

$$T_l(s, q^2 \neq m^2) = r_l(s, q^2 \neq m^2) T_l(s),$$

where $r_l(s, q^2)$ is a real-analytic function in s and has no elastic threshold singularity.

We also have to stress that for the "full" amplitude,

$$T(s, \cos\Theta, q^2) \Big|_{q^2 \neq m^2} = \frac{16\pi\sqrt{s}}{\sqrt{s-4m^2}} \sum_l (2l+1) T_{l*} P_l(\cos\Theta), \tag{4}$$

its phase generally depends on q^2-value.

It is a pleasure to thank A.Martin and T.T.Wu for useful discussions.